MAGNETIC-FIELD COMPRESSION BY A STRONG SHOCK WAVE

A. I. Bertinov, D. A. But, and V. I. Yudas

UDC 538.6

Compression of a magnetic field by a strong shock wave impinging on a solid wall is considered in a kinematic formulation. Variation of the magnetic field in the gap between the wall and the shock front is described by a Volterra integral equation which is solved numerically. The field distribution in gas and wall is obtained, as well as the dependence of the magnetic energy stored in them on the magnetic Reynolds number and the gas velocity in a coordinate system fixed in the shock front. Similar relations for the field and energy in the gap are also calculated. The results obtained are in good agreement with the data of other investigators.

To obtain large pulsed magnetic fields, one can use compression of a primary magnetic field by a strong shock or detonation wave under the condition the characteristic magnetic Reynolds number R_m is sufficiently large [1, 2].

We consider compression of a magnetic field by a strong shock wave incident on a solid conducting wall.

A general diagram of the model is shown in Fig. 1. The velocity of the shock front is w and the velocity and conductivity of the gas behind the front are, respectively, u and σ_1. The conductivity of the solid wall is σ_2, and its magnetic permeability is μ. The gas ahead of the wave is nonconducting. The surface of the wall and shock front are planes.

TABLE 1

v	R_{m1}: 25	50	100	300
0	0.92	0.96	0.98	0.994
0.1	0.92	0.96	0.98	0.992
0.25	0.84	0.92	0.96	0.990

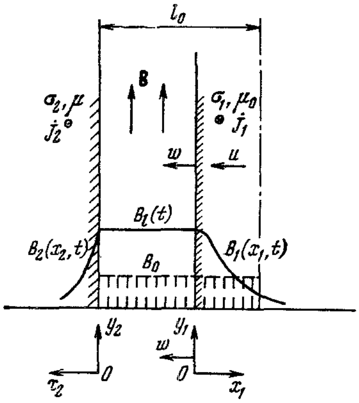

Fig. 1

The initial conditions for the magnetic field, which is parallel to the wall and the shock front, are: $B|_{t=0} = B_0$ over the length l_0 and $B|_{t=0} = 0$ outside the region l_0.

At $t = 0$, the shock front crosses the right boundary of the region l_0 and diffusion of the magnetic field behind the wave front and into the solid wall begins simultaneously. If the gas velocity u and the conductivities σ_1 and σ_2 are sufficiently large so that the magnitudes of the skin layers in the gas and wall are small in comparison with the initial gap l_0, the magnetic field B_1 between the wave front and the wall increases because of the currents j_1 behind the shock wave and j_2 in the wall.

We consider the penetration of the magnetic field behind the wave front in a coordinate system $x_1 0 y_1$ fixed in the front assuming $w = \text{const}$, $u = \text{const}$, and $\sigma_1 = \text{const}$. In the coordinate system $x_1 0 y_1$, the gas moves along x_1 at a velocity $v = w - u$. The initial and boundary conditions for the magnetic induction $B_1(x_1, t)$ behind the front are

$$B_1(x_1, 0) = 0,\ B_1(0,t) = B_l(t),\ B_l(0) = B_0 \tag{1}$$

where $B_l(t)$ is an unknown function.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 173-175, May-June, 1974. Original article submitted January 15, 1974.

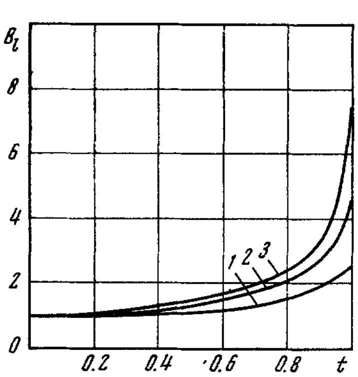

Fig. 2

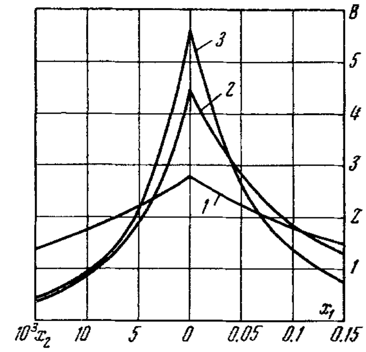

Fig. 3

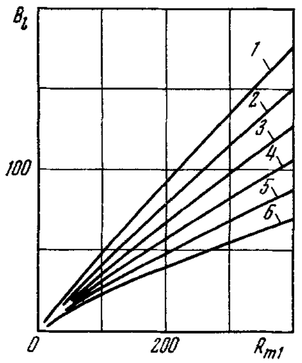

Fig. 4

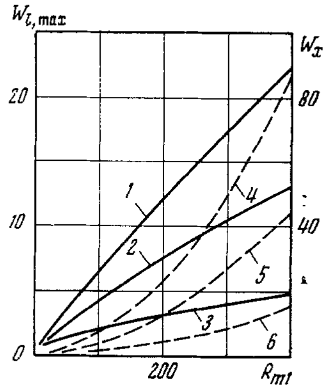

Fig. 5

In dimensionless variables, the equation of magnetic induction is

$$\frac{\partial B_1}{\partial t} = \frac{1}{R_{m1}} \frac{\partial^2 B_1}{\partial x_1^2} - v \frac{\partial B_1}{\partial x_1}, \qquad R_{m1} = \mu_0 \sigma_1 w l_0 \tag{2}$$

Here and in the following we select the quantities B_0, w, l_0, and l_0/w as scale factors.

The solution of Eq. (2) takes the form

$$B_1(x_1, t) = \frac{1}{2}\sqrt{\frac{R_{m1}}{\pi}} \exp\left[\frac{R_{m1}v}{2}\left(x_1 - \frac{1}{2}vt\right)\right] \times x_1 \int_0^t (t-\tau)^{-3/2} \exp\left[\frac{R_{m1}}{4}\left(v^2\tau - \frac{x_1^2}{t-\tau}\right)\right] B_l(\tau)\, d\tau \tag{3}$$

Solving the induction equation for the wall in the coordinate system $x_2 0 y_2$, we obtain

$$B_2(x_2, t) = \frac{1}{2}\sqrt{\frac{R_{m2}}{\pi}}\, x_2 \int_0^t (t-\tau)^{-3/2} \exp\left[-\frac{R_{m2}x_2^2}{4(t-\tau)}\right] B_l(\tau)\, d\tau \tag{4}$$

$$R_{m2} = \mu\sigma_2 w l_0$$

Thus a solution of the problem of field distribution reduces to a determination of $B_l(t)$.

To determine $B_l(t)$, we use the conservation of the total magnetic flux

$$\int_0^\infty B_1(x_1, t)\, dx_1 + \int_0^\infty B_2(x_2, t)\, dx_2 + B_l(t)\, l(t) = 1 \tag{5}$$

where $l(t)$ is the distance between the solid wall and the shock front in units of l_0.

Substituting Eqs. (3) and (4) into Eq. (5) and integrating, we arrive at a Volterra integral equation with respect to $B_l(t)$

$$B_l(t) + \frac{1}{1-t}\int_0^t \left\{\frac{1}{\sqrt{\pi R_{m2}(t-\tau)}} + \frac{v}{2}\left[1 + \operatorname{erf}(c) + \frac{\exp(-c^2)}{c\sqrt{\pi}}\right]\right\} B_l(\tau)\, d\tau = \frac{1}{1-t}, \qquad c = 0.5v\sqrt{R_{m1}(t-\tau)} \tag{6}$$

If $v=0$, Eq. (6) defines the magnetic field between converging solid walls under the initial conditions specified above [3]; a solution is given [4] for the case of a homogeneous initial condition $B=B_0$. With replacement in Eq. (6) of the first terms inside the integral sign by an expression similar to the second term and correction of the basis value of the velocity, Eq. (6) defines the field between two converging shockwaves of different intensities.

Equation (6) was solved numerically. The singularities occurring at $\tau=t$ and $t=1$ were removed by well-known methods [5]. The field distributions in the solid wall and gas were determined from Eqs. (3) and (4). The relative magnitude of the magnetic energy W_l stored in the gap l was determined from the expression $W_l=B_l^2(t)\, l(t)$; the value of the magnetic energy in the conducting media W_x was defined as

$$W_x=\int_0^\infty B_1^2(x_1,\ t)\,dx_1+\int_0^\infty B_2^2(x_2,\ t)\,dx_2$$

The calculations assumed $R_{m1}/R_{m2}=\sigma_1/\sigma_2=0.01$.

Figure 2 shows the increase of the field in the gap with time for $v=0.2$ and $R_{m1}=5$, 10, and 20 (curves 1, 2, and 3, respectively). Figure 3 shows the field distribution in the solid wall and in the gas behind the shock wave at the time $t=0.9$. For curve 1, $R_{m1}=10$, $v=0.1$; for curve 2, $R_{m1}=10^2$, $v=0.25$; for curve 3, $R_{m1}=10^2$, $v=0.1$. The dependence of the quantity B_l reached at $t=1$ on R_{m_1} and v is shown in Fig. 4. Curves 1-6 correspond to the values $v=0$, 0.05, 0.1, 0.15, 0.2, and 0.25. For large values of R_{m1}, the induction $B_l|_{t=1}$ is proportional to R_{m1}, which has been pointed out before [3].

Figure 5 shows the dependence of the maximum value $W_{l,max}$ of the magnetic energy in the gap (solid curves 1-3) and of the magnetic energy W_x in the solid wall and behind the shock wave at the time $t=1$ (dashed curves 4-6) on R_{m1} and v. Curves 1 and 4 correspond to $v=0$, curves 2 and 5 to $v=0.1$, and curves 3 and 6 to $v=0.25$. A comparison of the solid and dashed curves indicates that for $R_{m1}<100$, we roughly have $W_{l,\ max}>W_x(1)$, and for $R_{m1}>100$, we have $W_{l,\ max}<W_x(1)$. It is typical that the time t_m at which $W_{l,\ max}$ is reached also depends on R_{m1} and v (Table 1).

LITERATURE CITED

1. H. Knoepfel, Pulsed High Magnetic Fields,-Am. Elsevier (1970).
2. L. A. Zaklyaz'minskii, "Analysis of shock-wave reflection from the magnetic field of a current array," Zh. Prikl. Mekhan. Tekh. Fiz., No. 6 (1968).
3. E. I. Bichenkov, "Effect of finite conductivity on the production of strong magnetic fields by rapid compression of conducting shells," Zh. Prikl. Mekhan. Tekh. Fiz., No. 6 (1964).
4. A. Paton and W. Millar, "Compression of magnetic field between two semiinfinite slabs of constant conductivity," J. Appl. Phys., 35, No. 4 (1964).
5. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Advanced Analysis [in Russian], Fizmatgiz, Moscow—Leningrad (1962).